

## INVESTIGATION OF THE STATISTICAL CHARACTERISTICS OF WOLF NUMBERS RELIABLE SERIES: SIGNS OF SOLAR CYCLES LIKELIHOOD

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**Abstract:** In the course of the study the sunspots relative numbers authentic series and possible includes in the statistics of the reconstructed series individual solar cycles we studied the properties (statistical and observational) of individual 11-year solar cycles. The proximity of the averages for the duration of cycles is revealed, although the scatter is different in almost twice. More clearly this situation would be reflected for the ascending branch durations, both for averages and for dispersions. In depending on the maximum cycle size for the components of a reliable series the negative correlation between the duration of the rising and the value of the cycle maximum is revealed and its complete absence for the decay branch. And, on the contrary, for the restored series significant positive correlation between duration of a descending branch and size of a cycle maximum is manifested. Attempted to create a generalized portrait of an authentic solar cycle in order to obtain evidence-based description of the possible and impossible characteristics of individual solar cycles is made.

### 1. Introduction

The basic task of solar cycle's studies is the construction of the model of its changeability in the course of time, the development of the basic physical laws of those describing solar activity as a whole and, being based on the obtained results, the forecast of the subsequent solar cycles. For the correct solution of these problems it is necessary to be confident in the utilized reliable observant material and to know conditions and limits of the applicability the restored series of observations.

At present at our disposal there is

- a reliable (W1) series of average monthly values of the relative number of spots, obtained from the regular observations of several observatories since 1849 (cycles of solar activity 10 – 23);
- the numbered (W2) series of the numbers of Wolf, restored from the interrupted observations from 1749 until 1849 (Fig. 1);
- the restored average annual values of this index since 1700.

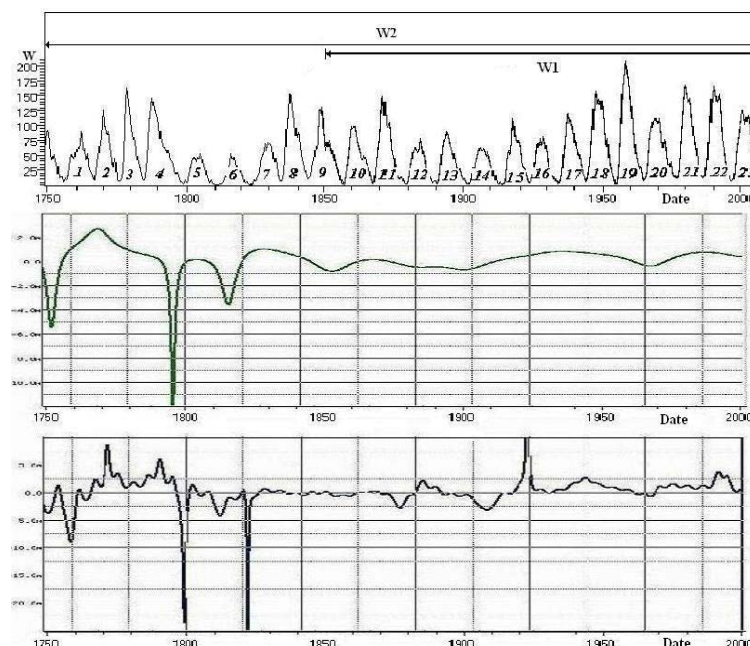


Fig.1. From top to bottom: The numbered (W2) and reliable (W1) series of the sunspots relative numbers;  $F[P2(W2)]$  – the trend of “instantaneous” frequency of W2 series component P2;  $F[P3(W2)]$  – the trend of “instantaneous” frequency of W2 component P3.

As shown in [1], the reliable and numbered series of the Wolf number have completely different spectral characteristics, and therefore the application of the latter for practical purposes leads to the appearance of artifacts and errors, and it can be used only for the rough estimates.

Reliable series includes complete fourteen cycles (from 10 through 23) and the calculated according to them average duration of the cycle  $T_{av} = 129.69$  months. The logarithm of the series  $W1$  power spectrum is represented in Fig. 2 (left upper angle), where are noted basic  $f^* = 0.007812$  1/month and multiple harmonics. Its fundamental period  $T^* = 1/f^* = 128.0$  months. On the basis of the spectrum nature, there is made the partition of signal into five spectral intervals, which correspond to the following temporary periods in the years: P1 [ $24 < T$ ], P2 [ $6.8 < T < 24$ ], P3 [ $4.26 < T < 6.8$ ], P4 [ $1.66 < T < 4.26$ ], P5 [ $T < 1.66$ ]. The overview of the signals ( $W1 = P1 + P2 + P3 + P4 + P5$ ) corresponding to them is represented in Fig. 2. The summation of series P1 and P2 reflects the fundamental temporary and amplitude characteristics of solar cycles. The component P3 corrects the branches of the rise and decrease. The component P4 transforms the smooth relief of solar cycles due to “quasi-biennale”. There is possible the appearance of local maximums, influence on the position of basic maximum and illegible manifestation of the solar cycle ends. By this means P3 and P4 give to solar cycles more individual nature. P5 – is high-frequency remainder, which includes annual and 155<sup>d</sup> harmonics. For the analysis of the chosen components it was used to the conversion of Gilbert [2]. The conversion makes it possible to remove uncertainty during the presence of envelope and phase of narrow-band signal, and on the smoothness “instantaneous” frequency to estimate the nature of process.

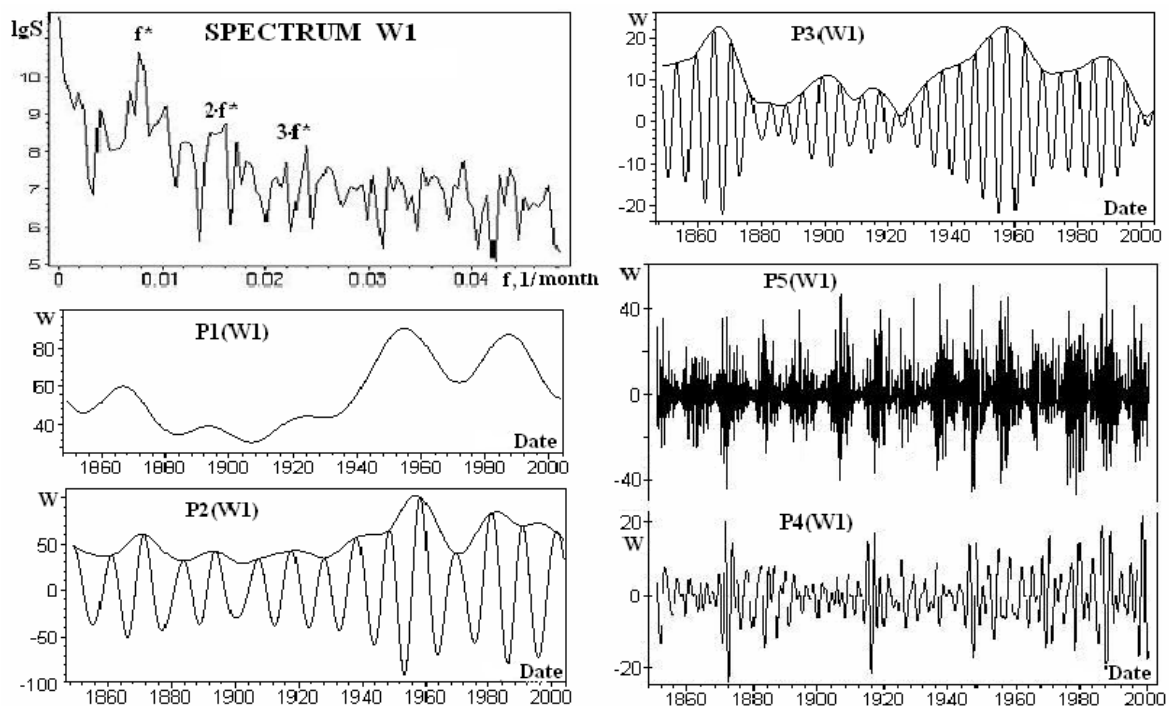


Fig. 2. Logarithm of the power spectrum of a reliable series  $W1$  is shown in left upper angle;  $P1(W1) - P5(W1)$  the component of a reliable series  $W1$ .

## 2. Some criteria of the solar cycle likelihood

Essential differences in the characteristics of the reliable (1849 – 2008) and restored (1749 – 1849) series of Wolf's number series have been noted in work [1] with the analysis of the basic spectral components smoothness, which form cycle and give its “*power engineering*”. Considerable disagreements remain also with the evaluation of the solar cycles integral characteristics [4], where the obtained criteria with the reliable solar cycles analysis are used for evaluating the restored solar cycles and are noted essential differences in the cycles II, IV, V and VI. Since ascending and descending branches represent different processes, it is reasonable to draw a comparison of their duration. On Fig. 3 is given the relation  $T_c/T_m$  for all cycles of the numbered series ( $T_c$  – the duration of solar cycle,  $T_m$  – the duration of the ascending branch, on axis X – the number of solar cycle).

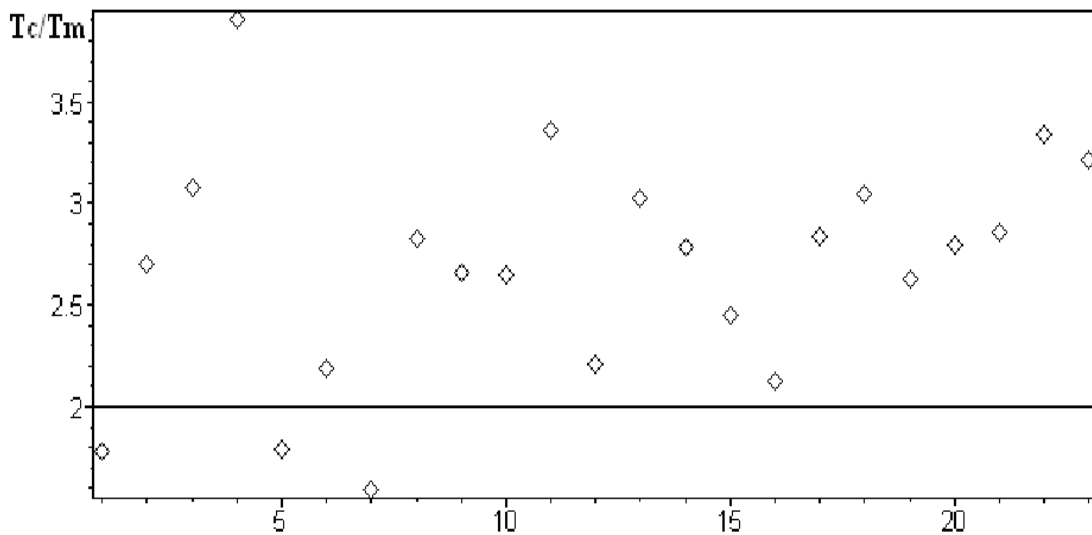


Fig. 3. Relation  $T_c/T_m$  for all cycles of the numbered series W2 ( $T_c$  – the duration of cycle,  $T_m$  – the duration of the rise branch, axis X – the number of cycle).

In all reliable cycles  $T_c/T_m > 2$ , i.e., the ascending branch is shorter than the descending branch. The cycles I, V and VII has the inverse ratio of the branches durations ( $T_c/T_m < 2$ ).

In Fig. 4 are represented dependences  $T_m$  and  $T_e = T_c - T_m$  from  $W_m$  – maximum value of the Wolf number in the cycle. Their correlations are given to the right above figures. Small crosses correspond to the restored cycles, circles – to reliable cycles. For the reliable cycles is visible the inverse correlation ( $-0.658$ ) between  $T_m$  and  $W_m$  (hence and Waldmeyer's rule [3]) and the absence of correlation ( $0.055$ ) between  $T_e$  and  $W_m$ .

Another nature these connections has for the restored cycles, where still the stronger inverse correlation ( $-0.898$ ) between  $T_m$  and  $W_m$  it passes into the positive ( $+0.466$ ) correlation  $T_e$  and  $W_m$ .

It is natural to continue the comparison of the general characteristics of cycles and to switch over to the statistical evaluation of their properties.

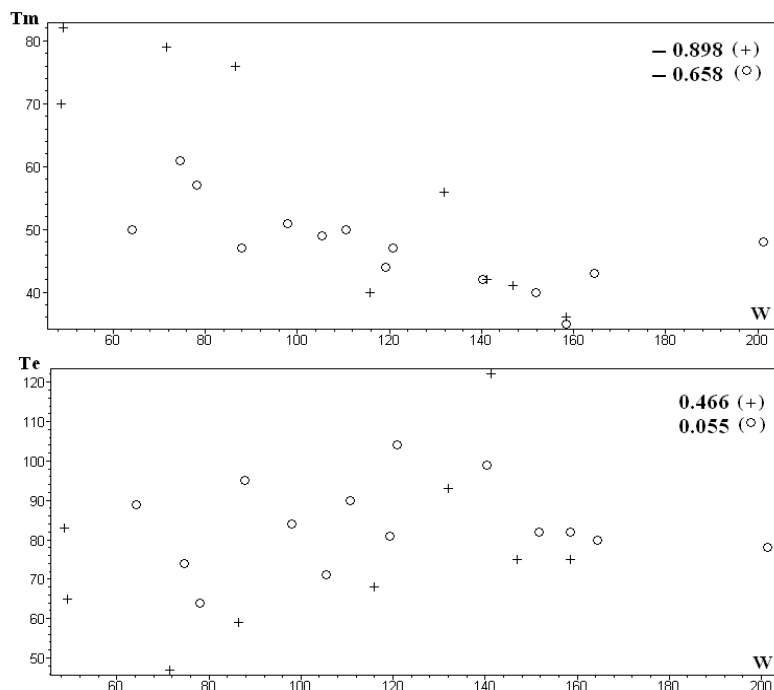


Fig. 4. Dependences of the ascending branches duration ( $T_m$ ) and the descending branches ( $T_e$ ) from  $W_m$  – maximum value of the Wolf number in the solar cycles.

### 3. General characteristics of solar cycles with their classification on the groups

In this part the comparison of the solar cycle's two group's characteristics is conducted: the group of solar cycles 1 – 9 and the group of solar cycles 10 – 23. The series of Wolf numbers, averaged on 13 months, is taken for their calculation. Each solar cycle is characterized by the following parameters:

T – duration of cycle;  
 Tm – duration of the branch of increase;  
 Wm – maximum value of the Wolf number in the cycle;  
 $sc = 2 \cdot Sc / (Wm \cdot Tc)$  – the standardized area of cycle;  
 $sm = 2 \cdot Sm / (Wm \cdot Tm)$  – the standardized area of the ascending branch;  
 skew – skewness (asymmetry) of solar cycle;  
 kurt – kurtosis (excess) factor.

Table 1 gives the mean, the root from the dispersion ( $\sigma^{1/2}$ ) and their relation ( $\sigma^{1/2}/\text{mean}$ ) for each of seven parameters, but calculated according to the solar cycle's groups. Upper value corresponds to the first group (1 – 9), it is lower (bold) for solar cycles 10 – 23.

Table 1.

	Mean	$\sigma^{1/2}$	$\sigma^{1/2}/\text{mean}$
Tc	134.33 <b>131.21</b>	19.00 <b>10.07</b>	0.141 <b>0.077</b>
Tm	58.00 <b>47.43</b>	17.77 <b>6.41</b>	0.306 <b>0.135</b>
Wm	105.58 <b>119.66</b>	40.16 <b>38.02</b>	0.380 <b>0.318</b>
sc	0.91 <b>0.95</b>	0.09 <b>0.08</b>	0.099 <b>0.088</b>
sm	0.90 <b>0.96</b>	0.16 <b>0.14</b>	0.182 <b>0.150</b>
skew	0.26 <b>0.18</b>	0.22 <b>0.15</b>	0.836 <b>0.874</b>
kurt	1.83 <b>1.58</b>	0.28 <b>0.19</b>	0.154 <b>0.119</b>

The close proximity of the means for Tc is visible from the table 1, but spread is different almost two times. Situation for the means and the dispersion Tm is still more contrasting. Are close these estimations, also for Wm. As a whole it is possible to note that the characteristic of the second group better or it is considerable (Tc, Tm) better.

Useful qualitative assessments according to the groups give histograms (Fig. 5), let us give them for the the ascending branch parameters. In cycles 10 – 23 are substituted the normal (for Tm) and uniform (for sm) distributions, calculated according to the solar cycles characteristics. Estimation of normal distribution sm for cycles 1–9 is also given.

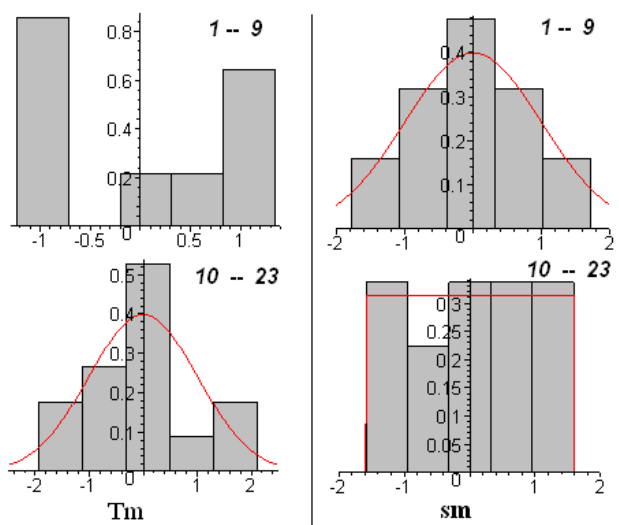


Fig. 5. Histograms for the parameters of the ascending branches and their given areas for the reliable and restored series of the Wolf numbers.

It seems that it suffices problematically to connect “torn” and smooth distributions of the first group. Following Fig. 6 demonstrate difference in the nature of the solar cycle groups structuring. Large ordering is inherent in the second group of solar cycles.

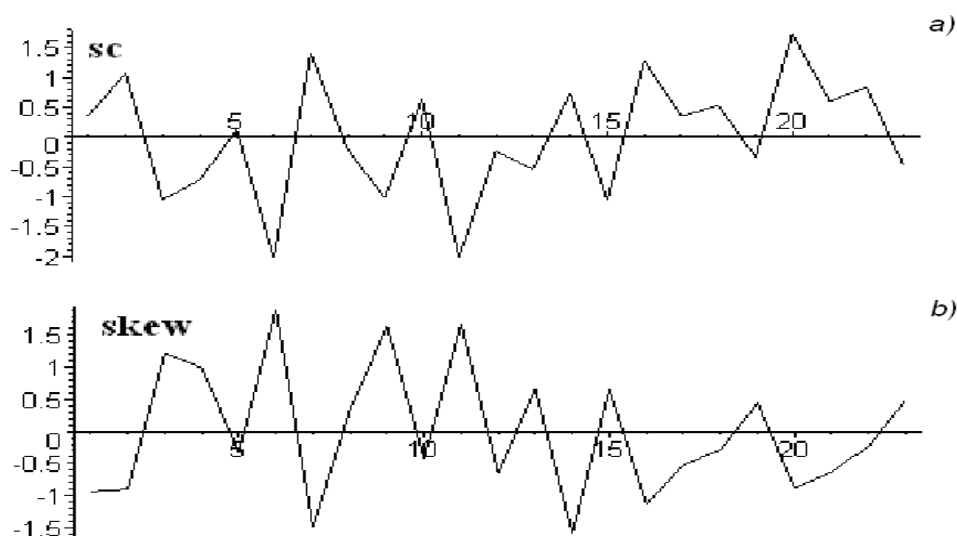


Fig. 6. Changes in the standardized areas of the ascending branches *sc* (a) and degree of the solar cycles skewness – asymmetry (b), axis X – the number of cycle.

### 3. Conclusion

In the work the attempt to compare the properties of the Wolf numbers reliable and restored series and characteristic of the solar cycles with different approaches corresponding to them are made. Thus there are far into “the zone” of the smallest criticism fell three solar cycles from the restored series – III, VIII, IX. On the basis of the criteria, inherent in reliable cycles, it is possible to speak about the essential distortions of the Wolf number values during their reconstruction.

A significant quantity of publications is devoted to the restoration of solar data according to the indirect characteristics. The analysis of the fractal properties of a number of eleven sequoias annual rings widths is represented in the work [5]. Is noted the isolation of the temporary moments of those coinciding with the Sперer and Maunder minimums of the solar activity. Dalton's minimum, in this approach, is not manifested.

### References:

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